Lecture 3/20/2017

• Review of Kirchhoff’s Rules
• Review of emf in Parallel and in Series
• Charging Capacitors
• Discharging Capacitors
In the circuit below, what is the current through $R_1$?

1) 10 A  
2) zero  
3) 5 A  
4) 2 A  
5) 7 A
What is the current in branch P?

1) 2 A
2) 3 A
3) 5 A
4) 6 A
5) 10 A
The lightbulbs in the circuit are identical. When the switch is closed, what happens?

1) both bulbs go out
2) intensity of both bulbs increases
3) intensity of both bulbs decreases
4) A gets brighter and B gets dimmer
5) nothing changes ($R_A = R_B$)
Some circuits cannot be broken down into series and parallel connections. For these circuits we use Kirchhoff’s rules.
Kirchhoff’s Rules

Junction rule: The sum of currents entering a junction equals the sum of the currents leaving it.

\[ \sum_{in} I = \sum_{out} I \]
Loop rule: The sum of the changes in potential around a closed loop is zero.

\[ \sum_{\text{Closed Loop}} \Delta V = 0 \]
Which of the equations is valid for the circuit below?

1) $2 - I_1 - 2I_2 = 0$
2) $2 - 2I_1 - 2I_2 - 4I_3 = 0$
3) $2 - I_1 - 4 - 2I_2 = 0$
4) $I_3 - 4 - 2I_2 + 6 = 0$
5) $2 - I_1 - 3I_3 - 6 = 0$
EMFs in series in the same direction: total voltage is the sum of the separate voltages.
EMFs in series, opposite direction: total voltage is the difference, but the lower-voltage battery is charged.
EMFs in parallel only make sense if the voltages are the same; this arrangement can produce more current than a single emf.
When the switch is closed, the capacitor will begin to charge. As it does, the voltage across it increases, and the current through the resistor decreases.
Charging Capacitor

To find the voltage as a function of time, we write the equation for the voltage changes around the loop:

\[ \mathcal{E} = IR + \frac{Q}{C} - \frac{(q - \mathcal{E}C)}{RC} = \frac{dq}{dt} \]

Since \( Q = dI/dt \), we can integrate to find the charge as a function of time:

\[ -\frac{dt}{RC} = \frac{dq}{q - \mathcal{E}C} \quad \text{and} \quad -\left. \frac{t}{RC} \right|_0^t = \ln(q - \mathcal{E}C) \bigg|_0^Q \]

\[ Q(t) = C\mathcal{E}(1 - e^{-t/RC}) \]
Circuits Containing Resistor and Capacitor (RC Circuits)

The voltage across the capacitor is $V_C = Q/C$:

$$V_C = \varepsilon(1 - e^{-t/RC}).$$

The quantity $RC$ that appears in the exponent is called the time constant of the circuit:

$$\tau = RC.$$
Charging Capacitor

The current at any time \( t \) can be found by differentiating the charge:

\[
I(t) = \frac{dQ(t)}{dt} = \frac{d}{dt} C \mathcal{E} \left(1 - e^{-t/RC}\right)
\]

\[
I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}
\]
If an isolated charged capacitor is connected across a resistor, it discharges:

\[ Q(t) = Q_0 e^{-t/RC} \]
Discharging Capacitor

Once again, the voltage and current as a function of time can be found from the charge:

\[ V_C = V_0 e^{-t/RC} \]

and

\[ I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}. \]
Three resistors of values 2 Ω, 6 Ω and 12 Ω are connected across a DC voltage source as shown. If the total current through the circuit is I = 5.0 A, what is the current through the 12 Ω resistor?

A) 0.6 A
B) 0.7 A
C) 0.5 A
D) 0.9 A
E) 0.8 A
The values of the resistances are 50 Ω for resistor A, 100Ω for resistor B, and 300 Ω for resistor C. The battery has a voltage $V = 5$ V. What is the power delivered by the battery?

A) 2.5 W  
B) 1.5 W  
C) 0.15  
D) 0.45 W  
E) 0.25 W
Extra Credit Exam#1

An uncharged capacitor $C=5.00\mu F$ and a resistor $R=8.00 \times 10^5 \, \Omega$ are connected in series to a 12.0 V battery. How long will it take for the capacitor to charge to 90% capacity?

Charging Capacitor

A) 9.2 s  
B) 11.8 s  
C) 0.3 s  
D) 4.0 s  
E) 3.6 s