Lecture 4/3/2017

• Review: Force between two parallel wires.
• Review: application of Biot-Savart law.
• Ampère’s Law
• Magnetic field of a solenoid
• Magnetic Materials
Force between Two Parallel Wires

The magnetic field produced at the position of wire 2 due to the current in wire 1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}.$$  

The force this field exerts on a length $\ell_2$ of wire 2 is

$$F_2 = \frac{\mu_0 I_1 I_2}{2\pi d} \ell_2.$$ [parallel wires]
Consider a thin, straight wire carrying a constant current $I$ and placed along the x axis. Determine the magnitude and direction of the magnetic field at point P due to this current.

$$d\vec{B}(0, a) = \frac{\mu_o I ds \times \hat{r}}{4\pi r^2}$$

$$\vec{B}(0, a) = \int_{-\infty}^{\infty} \frac{\mu_o I ds \times \hat{r}}{4\pi r^2}$$

$$\vec{B}(0, a) = \frac{\mu_o I}{4\pi} \int_{-\infty}^{\infty} \frac{dx \sin \theta}{r^2} \hat{k}$$

$$r = \frac{a}{\sin \theta} \quad a = -acot\theta \quad dx = a \csc^2 \theta \, d\theta$$

$$B(0, a) = \frac{\mu_o I}{4\pi} \int_{\theta_1}^{\theta_2} \csc^2 \theta \sin \theta \, d\theta = \frac{\mu_o I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta = \frac{\mu_o}{4\pi a} (\cos \theta_1 - \cos \theta_2)$$

$$\lim_{x \to -\infty} \cos \theta_1 = 1 \quad \lim_{x \to +\infty} \cos \theta_2 = -1 \quad \Rightarrow B(r) = \frac{\mu_o}{2\pi r} \text{ for an infinitely long wire}$$
Ampère’s Law

Ampère’s law relates the magnetic field around a closed loop to the total current flowing through the loop:

\[ \oint B \cdot d\ell = \mu_0 I_{enclosed} \]

This integral is taken around the edge of the closed loop.
Using Ampère’s law to find the field around a long straight wire:

\[
\mu_0 I = \oint \vec{B} \cdot d\vec{\ell} \\
= \oint \vec{B} \cdot d\vec{\ell} \\
= B \oint d\ell \quad \text{(constant } B \text{ and angle between } \vec{B} \text{ and } d\vec{\ell} \text{ is constant)} \\
= B(2\pi r) \\
\Rightarrow B(r) = \frac{\mu_0}{2\pi r}
\]
Solving problems using Ampère’s law:

- Ampère’s law is only useful for solving problems when there is a great deal of symmetry. Identify the symmetry.

- Choose an integration path that reflects the symmetry (typically, the path is along lines where the field is constant and perpendicular to the field where it is changing).

- Use the symmetry to determine the direction of the field.

- Determine the enclosed current.
$I_1 = 5.00\,\text{A}$ and the wire lies in the plane of the rectangular loop with current $I_2 = 10.0\,\text{A}$. The dimensions are $c = 0.100\,\text{m}$, $a = 0.150\,\text{m}$ and $\ell = 0.450\,\text{m}$. Find the direction of the net force exerted on the loop by the magnetic field created by the wire.

a) Out of the page  
b) Into the page  
c) Left  
d) Right  
e) Up  
f) Down
Two long parallel wires each having a mass per unit length of 40g/m are supported in a horizontal plane by strings 6.00 cm long. When both wires carry the same current I, the wires repel each other so that the angle between them is 16.0°. The currents are:

A) In the same direction.
B) In opposite directions.
C) Cannot be determined.
A solenoid is a coil of wire containing many loops. To find the field inside, we use Ampère’s law along the path indicated in the figure.
The field is zero outside the solenoid, and the path integral is zero along the vertical lines, so the field is:

\[ B = \mu_0 nI \]

\( n \) is the number of loops per unit length.
Ferromagnetic materials are those that can become strongly magnetized, such as iron and nickel.

These materials are made up of tiny regions called domains; the magnetic field in each domain is in a single direction.
When the material is unmagnetized, the domains are randomly oriented. They can be partially or fully aligned by placing the material in an external magnetic field.
A magnet, if undisturbed, will tend to retain its magnetism. It can be demagnetized by shock or heat.

The relationship between the external magnetic field and the internal field in a ferromagnet is not simple, as the magnetization can vary.
Magnetic Fields in Magnetic Materials;

If a ferromagnetic material is placed in the core of a solenoid or toroid, the magnetic field is enhanced by the field created by the ferromagnet itself.

This is usually much greater than the field created by the current alone.

If we write

$$B = \mu I$$

where $\mu$ is the magnetic permeability, ferromagnets have $\mu >> \mu_0$, while all other materials have $\mu \approx \mu_0$. 