CLARIFICATIONS ON FLUX AND AREA VECTOR

The purpose of this pre-recitation is for you to become thoroughly acquainted with the concepts of flux. We will also discuss the concept of an area vector. You know how to calculate the area of a rectangle or a square or a circle. However, in this pre-recitation we want to describe to you how, by defining area as a vector (that is, by giving area a mathematically convenient direction in addition to its magnitude), we can make the descriptions of flux and other important physical quantities simpler and more convenient.

Consider the following scenario where water is gushing out of a faucet and being collected into a metal bucket with rectangular opening. This particular physical setup is also mentioned in your lecture (Lecture #3) and you may also want to consult Chapter 12.4 from Physics 124, last year.

A. The three situations in Figure 1 show three distinct orientations of the bucket. Note in all three scenarios the three faucets are identical and so the velocity of the water jets coming out of the faucets are also equal, as shown by equal blue velocity vectors. The areas of the cross sections of the top of the bucket are also identical in size. The amount of water that flows through a particular area in a fixed time (for example, 1 second) is known as the fluid “flux” through that area. Now answer the following questions:

1. Initially all of the buckets are dry. Rank the three buckets in order of water collected after 1 minute, from least to greatest.

2. Is the most water collected when $\vec{A}$ and $\vec{v}$ are parallel or when they are perpendicular? In which situation is the least water collected?

It is convenient to define an area vector $\vec{A}$ as a vector whose magnitude is equal to the area of a particular surface and whose direction is perpendicular to the surface. We redraw the situation from Figure 1 with the area vector included, as shown to the right in Figure 2.

2. Is the most water collected when $\vec{A}$ and $\vec{v}$ are parallel or when they are perpendicular? In which situation is the least water collected?
B. Consider Figure 3, where we have three different faucets from where water is streaming out into three buckets. The buckets were initially dry and have same cross-sectional area $A$. The velocities of water jets that are coming out of three faucets are different and the magnitudes of the velocities are proportional to the lengths of the blue arrows. Now answer the following questions:

1. Rank the three buckets in order of water collected, from least to greatest, after 1 minute.

2. Does the total amount of water collected in the buckets after 1 minute increase or decrease as we increase the speed of the water jet out of the faucets?

C. Consider Figure 4, where all of the three faucets are identical and the velocities of the water jets out of them are also equal. We now have three different buckets with three different cross-sectional area openings put below the three faucets. The buckets, as before, were initially dry. Now answer the following questions:

1. Rank the three buckets in order of water collected, least to greatest, after 1 minute.

2. Does the water collected in the buckets increase or decrease as we increase the cross-sectional areas of the bucket openings?
D. Calculating fluid flux involves multiplying two vectors. We need to consider how to go about multiplying quantities that have both direction and magnitude. \( \mathbf{A} \) and \( \mathbf{v} \) are not always parallel to each other; as you saw in part A we have to consider their orientations with respect to each other. There are two ways of multiplying vectors together, either the dot product (which is a maximum when the vectors are parallel) or the cross product (which is a maximum when the vectors are perpendicular.)

Given the conclusions obtained in part A and the discussion above about dot and cross product, which of the following equations correctly describes fluid flux?

\[
\text{Fluid Flux } \propto \mathbf{v} \cdot \mathbf{A}, \quad \text{or} \quad \text{Fluid Flux } \propto \mathbf{v} \times \mathbf{A}.
\]

E. Extending the idea of water or fluid flux to electric flux is straightforward. All you must do is replace water jets from the faucets with electric field lines produced by distributions of source charges. Instead of looking at the amount of water collected in the buckets, you look at how many electric field lines cross a particular area to determine the electric flux through that area. Mathematically, replace the velocity vector (\( \mathbf{v} \)) with the electric field vector (\( \mathbf{E} \)), and instead of a proportionality it becomes an equality.

Given the above discussion about electric flux, rank the three situations shown in Figure 5 in order of least to greatest electric flux through area \( \mathbf{A} \).

Figure 5