Lecture 2  June 1, 2017

• Review: Adding Vectors by Components
• Reference Frames and Displacement
• Average Velocity
• Instantaneous Velocity
• Acceleration
• Motion at Constant Acceleration
• Solving Problems
• Freely Falling Objects
• Graphical Analysis of Linear Motion
Vectors and Scalars

A vector has magnitude as well as direction.

Some vector quantities: displacement, velocity, force, momentum

A scalar has only a magnitude.

Some scalar quantities: mass, time, temperature
Adding Vectors by Components

If the components are perpendicular, they can be found using trigonometric functions.

\[
\sin \theta = \frac{V_y}{V} \\
\cos \theta = \frac{V_x}{V} \\
\tan \theta = \frac{V_y}{V_x} \\
V^2 = V_x^2 + V_y^2
\]
Adding Vectors by Components

The components are effectively one-dimensional, so they can be added arithmetically:

\[ \mathbf{V}_R = \mathbf{V}_1 + \mathbf{V}_2 \]
Let $\vec{V}_1 = 40.0 \, m, 30.0^\circ$
and
$\vec{V}_2 = 20.0 \, m, 60.0^\circ$

Write $\vec{V}_1$ and $\vec{V}_2$ in components (i.e. vectors parallel to the x, and y axis).

**Using the Pythagorean theorem:**

$\vec{V}_{1x} = 40 \cos(30^\circ) = 34.6 \, m, 0^\circ$

$\vec{V}_{1y} = 40 \sin(30^\circ) = 20.0 \, m, 90^\circ$

$\vec{V}_{1x} = 20 \cos(60^\circ) = 10.0 \, m, 0^\circ$

$\vec{V}_{1x} = 20 \sin(60^\circ) = 17.3 \, m, 90^\circ$
Adding Vectors by Components

Using the Pythagorean theorem:

\[
\vec{V}_1 = \vec{V}_{1x} + \vec{V}_{1y} = 34.6 \, m, 0^\circ + 20.0 \, m, 90^\circ
\]

\[
\vec{V}_2 = \vec{V}_{2x} + \vec{V}_{2y} = 10.0 \, m, 0^\circ + 17.3 \, m, 90^\circ
\]

Then:

\[
\vec{V}_R(m) = \vec{V}_1 + \vec{V}_2 = (34.6 + 10.0), 0^\circ + (20.0 + 17.3), 90^\circ
\]

\[
\vec{V}_R(m) = \vec{V}_1 + \vec{V}_2 = (44.6), 0^\circ + (37.3), 90^\circ
\]
Adding Vectors by Components

\[ \vec{V}_R(m) = \vec{V}_1 + \vec{V}_2 = (34.6 + 10.0), 0^\circ + (20.0 + 17.3), 90^\circ \]

\[ \vec{V}_R(m) = \vec{V}_1 + \vec{V}_2 = (44.6), 0^\circ + (37.3), 90^\circ \]

Let's define 2 vectors:

\( \hat{x} \) and \( \hat{y} \) where \( |\hat{x}| = |\hat{y}| = 1 \) and

\( \hat{x} = \text{Parallel to the } x - \text{axis (e.g. } 0^\circ) \),

\( \hat{y} = \text{Parallel to the } y - \text{axis (e.g. } 90^\circ) \)

Then:

\[ \vec{V}_1(m) = 34.6\hat{x} + 20.0\hat{y} \]

\[ \vec{V}_2(m) = 10.0\hat{x} + 17.3\hat{y} \]

And

\[ \vec{V}_R = 44.6\hat{x} + 37.3\hat{y} \]

\[ |\vec{V}_R| = \sqrt{44.6^2 + 37.3^2} = 58.1 \text{m} \]

\[ \theta_R = \arctan(\frac{37.3}{44.6}) = 39.9^\circ \]
Describing Motion: Kinematics in One Dimension
Reference Frames and Displacement

Any measurement of position, distance, or speed must be made with respect to a reference frame.

For example, if you are sitting on a train and someone walks down the aisle, their speed with respect to the train is a few miles per hour, at most. Their speed with respect to the ground is much higher.
Reference Frames and Displacement

We make a distinction between distance and displacement.

**Displacement** (blue line) is how far the object is from its starting point, regardless of how it got there.

**Distance** traveled (dashed line) is measured along the actual path.
Reference Frames and Displacement

The displacement (in any 1D) is written: \( \Delta x = x_2 - x_1 \)

- Left: Displacement is positive.
- Right: Displacement is negative.
**Speed vs. Velocity and Average Velocity**

**Speed**: how far an object travels in a given time interval

\[
\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}
\]

**Velocity** includes directional information:

\[
\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{time elapsed}}
\]
Instantaneous Velocity

The instantaneous velocity is the average velocity, in the limit as the time interval becomes infinitesimally short.

\[ \nu = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \]

Graphs showing a constant velocity

Graph showing a varying velocity
**Acceleration**

**Acceleration** is the rate of change of velocity.

\[
\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}
\]

- **At** \( t_1 = 0 \)
  - \( v_1 = 0 \)
  - \( a = 15 \text{ km/h/s} \)

- **At** \( t = 1.0 \text{ s} \)
  - \( v = 15 \text{ km/h} \)

- **At** \( t = 2.0 \text{ s} \)
  - \( v = 30 \text{ km/h} \)

- **At** \( t = t_2 = 5.0 \text{ s} \)
  - \( v = \dot{v}_2 = 75 \text{ km/h} \)
Acceleration is a vector, although in one-dimensional motion we only need the sign.

The previous image shows positive acceleration; here is negative acceleration:

\[
\begin{align*}
\text{at } t_1 &= 0 \\
v_1 &= 15.0 \text{ m/s} \\
a &= -2.0 \text{ m/s}^2
\end{align*}
\]

\[
\begin{align*}
\text{at } t_2 &= 5.0 \text{ s} \\
v_2 &= 5.0 \text{ m/s}
\end{align*}
\]
There is a difference between negative acceleration and deceleration:

**Negative acceleration** is acceleration in the negative direction as defined by the coordinate system.

**Deceleration** occurs when the acceleration is opposite in direction to the velocity.
The *instantaneous* acceleration is the average acceleration, in the *limit* as the time interval becomes *infinitesimally* short.

\[ a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \]
Special Case:
Motion at Constant Acceleration
Special Case:
Motion at Constant Acceleration

The average velocity of an object during a time interval \( t \) is

\[
\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}
\]

The acceleration, assumed constant, is

\[
a = \frac{v - v_0}{t}.
\]
Special Case: Motion at Constant Acceleration

In addition, as the velocity is increasing at a constant rate, we know that

$$\ddot{v} = \frac{v_0 + \dot{v}}{2}.$$ 

Combining these last three equations, we find:

$$x = x_0 + \ddot{v}t$$

$$= x_0 + \left(\frac{v_0 + \dot{v}}{2}\right)t$$

$$= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t$$

or

$$x = x_0 + v_0 t + \frac{1}{2}at^2.$$
We can also combine these equations so as to eliminate $t$:

$$v^2 = v_0^2 + 2a(x - x_0)$$

We now have all the equations we need to solve constant-acceleration problems:

1. $v = v_0 + at$
2. $x = x_0 + v_0 t + \frac{1}{2}at^2$
3. $v^2 = v_0^2 + 2a(x - x_0)$
4. $\ddot{v} = \frac{v + v_0}{2}$
Solving Problems

1. Read the whole problem and make sure you understand it. Then read it again.
2. Decide on the objects under study and what the time interval is.
3. Draw a diagram and choose coordinate axes.
4. Write down the known (given) quantities, and then the unknown ones that you need to find.
6. Which equations relate the known and unknown quantities? Are they valid in this situation? Solve algebraically for the unknown quantities, and check that your result is sensible (correct dimensions).
7. Calculate the solution and round it to the appropriate number of significant figures.
8. Look at the result—is it reasonable? Does it agree with a rough estimate?
9. Check the units again.
Freely Falling Objects

Near the surface of the Earth, all objects experience the same acceleration due to gravity.

This is one of the most common examples of motion with constant acceleration.
Freely Falling Objects

In the absence of air resistance, all objects fall with the same acceleration, although this may be hard to tell by testing in an environment where there is air resistance.
The acceleration due to gravity at the Earth’s surface is approximately \(9.80\, \text{m/s}^2\).
Graphical Analysis of Linear Motion

This is a graph of $x$ vs. $t$ for an object moving with constant velocity. The velocity is the slope of the $x$-$t$ curve.
Example:

A speeder passes a stationary police car while traveling at a constant speed of 0.094m/s.
If a police car starts to move with an instantaneous and constant speed of 0.389m/s and it reacts 2s after seeing the speeder pass him, how long does it take to catch on to the speeder?

1. We identify this as **motion under constant acceleration** then we use the solution:

   \[ x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \]
   \[ v_x(t) = v_{0x} + a_x t \]
   \[ v_x^2 = v_0^2 + 2a_x \Delta x \]

   where \( \Delta x = x - x_0 \)

   (\( x_0 \) is the position at the chosen initial time)
Example:

A speeder passes a stationary police car while traveling at a constant speed of 0.094 m/s.

If a police car starts to move with an instantaneous and constant speed of 0.389 m/s and it reacts 2 s after seeing the speeder pass him, how long does it take to catch on to the speeder?

2. Letting $t=0$ be the time when the police car first starts its motion and the origin of our coordinate system at the point where the police car is at $t=0$. Then we can model the position over time for the speeder $x(t)_s$ and the police car $x(t)_p$ as:

$$x(t)_s = 0.188 + 0.094 t \quad \text{and} \quad x(t)_p = 0 + 0.389 t$$

3. We want to find the time when the positions of the speeder and the police care are the same (i.e. $x_s = x_p$)

$$0.188 + 0.94t = 0.389t \quad \rightarrow \quad t = 0.64s$$