Lecture 13, 6/27/2017

• Review: Elasticity; Stress and Strain
• Review: Pascal and Archimedes' Principles
• Fluid Dynamics: Equation of continuity, Bernoulli's Equation
• Temperature and Heat
• Thermal Expansion
• Review: Absolute temperature (Kelvin)
• Specific Heat and Calorimetry
Quick Review

- **Hooke’s Law** \( \vec{F} = -k\vec{x} \) where \( k \) is the spring’s constant

- **stress** = \( \frac{\text{force}}{\text{area}} = \frac{F}{A} \)  \[ \text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta \ell}{\ell_0} \]
  - In tensile stress, forces tend to stretch the object.
  - Compressional stress is exactly the opposite of tensional stress.

- The (mass) density \( \rho \) of an object is its mass per unit volume: \( \rho = \frac{m}{V} \)

- **Pressure** \( P = \frac{F}{A} \)  \( F \) is the magnitude of the force acting perpendicular to the area \( A \).
  - Pressure is a scalar. The SI units of pressure are pascals (1Pa=1N/m^2)
Variation of Pressure with Depth

Consider a liquid of uniform density $\rho$.

- Select a sample of the liquid within an imaginary cylinder of cross-sectional area $A$ extending from depth $d$ to $d+h$.
- The liquid external to our sample exerts forces at all points perpendicular to the surface of the sample.
- The pressure exerted by the liquid on the top face of the sample is $P_0$ and on the bottom face is $P$.
  - Then the upward force exerted by the outside fluid on the bottom of the cylinder has as magnitude $PA$ (recall $P=F/A$) and the downward force on the top face is $P_0A$
  - The mass of the liquid in the cylinder is $m = \rho V = \rho Ah$ (recall $\rho = \frac{m}{V}$)

If $\rho$ is constant the liquid is incompressible.
Variation of Pressure with Depth

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- Then the upward force exerted by the outside fluid on the bottom of the cylinder has as magnitude $PA$ (recall $P=F/A$) and the downward force on the top face is $P_0A$.
- The mass of the liquid in the cylinder is
  \[
  m = \rho V = \rho Ah \quad (recall \ \rho = \frac{m}{V})
  \]

Choosing Upward as positive we see that:

\[
\Sigma \hat{F} = PA\hat{y} - P_0A\hat{y} - Mg\hat{y} = 0
\]

\[
\Rightarrow PA - P_0A - \rho Ahg = 0
\]

\[
\Rightarrow PA - P_0A = \rho Ahg
\]

\[
\Rightarrow P = P_0 + \rho gh
\]

$P = \rho gh$ pressure below the surface.
Pascal’s Principle

If an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount. \( P_{in} = P_{out} \)

You exert force on a piston with circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston with a radius of 5.0 m. What force must you apply to lift a car weighing 13.3 kN?

\[
\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \rightarrow F_{in} = F_{out} \frac{A_{in}}{A_{out}}
\]

\[
F_{in} = 13300 \frac{\pi(0.05m)^2}{\pi(5.0m)^2} = 1.33N
\]
Fluids in Motion; Flow Rate and the Equation of Continuity

If the flow of a fluid is smooth, it is called streamline or laminar flow (a).

Above a certain speed, the flow becomes turbulent (b). Turbulent flow has eddies; the viscosity of the fluid is much greater when eddies are present.
Equation of Continuity

We will deal with laminar flow.

The mass flow rate is the mass that passes a given point per unit time. The flow rates at any two points must be equal, as long as no fluid is being added or taken away.

This gives us the equation of continuity:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$
Fluids in Motion; Flow Rate and the Equation of Continuity

If the density doesn’t change—typical for liquids—this simplifies to $A_1 v_1 = A_2 v_2$.

→ Where the pipe is wider, the flow is slower.
Bernoulli’s Equation

A fluid can also change its height.

By looking at the work done as it moves, we find:

$$P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 = P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1.$$  

This is Bernoulli’s equation. One thing it tells us is that as the speed goes up, the pressure goes down.
Using Bernoulli’s principle, we find that the speed of fluid coming from a spigot on an open tank is:

\[ \frac{1}{2} \rho v_1^2 + \rho g y_1 = \rho g y_2 \]

or

\[ v_1 = \sqrt{2g(y_2 - y_1)}. \]

This is called Torricelli’s theorem.
Applications of Bernoulli’s Principle: Airplanes

Lift on an airplane wing is due to the different air speeds and pressures on the two surfaces of the wing.
Applications of Bernoulli’s Principle: Torricelli, Airplanes, Baseballs, Blood Flow

A ball’s path will curve due to its spin, which results in the air speeds on the two sides of the ball not being equal.
A person with constricted arteries will find that they may experience a temporary lack of blood to the brain as blood speeds up to get past the constriction, thereby reducing the pressure.
Temperature and Heat
On a microscopic scale, the arrangements of molecules in solids (a), liquids (b), and gases (c) are quite different.
Temperature and Thermometers

**Temperature** is a measure of how hot or cold something is.

Most materials expand when heated.
Temperature and Thermometers

Thermometers are instruments designed to measure temperature. In order to do this, they take advantage of some property of matter that changes with temperature.

Early thermometers:
Common thermometers used today include the liquid-in-glass type and the bimetallic strip.
Temperature and Thermometers

Temperature is generally measured using either the Fahrenheit or the Celsius scale.

The freezing point of water is 0°C, or 32°F; the boiling point of water is 100°C, or 212°F.
Thermal Equilibrium and the Zeroth Law of Thermodynamics

Two objects placed in thermal contact will eventually come to the same temperature. When they do, we say they are in thermal equilibrium.

The zeroth law of thermodynamics says that if two objects are each in equilibrium with a third object, they are also in thermal equilibrium with each other.
Linear expansion occurs when an object is heated.

\[ \ell = \ell_0(1 + \alpha \Delta T), \]

Here, \( \alpha \) is the coefficient of linear expansion.
Example

A segment of steel railroad track has a length of 30.000 m when the temperature is 0.0°C. What is its length when the temperature is 40.0°C? The linear expansion coefficient of steel is: \( \alpha_{steel} = 11 \times 10^{-6} /°C \)

\[
\ell = \ell_0 (1 + \alpha \Delta T) \implies \Delta \ell = \ell_0 \alpha \Delta T
\]

\[
\Delta \ell = 30.000m \cdot (11 \times 10^{-6} /K) \cdot 40K = 0.0132m
\]

\[
\ell = 30.000m + 0.0132m = 30.0132m
\]
Volume expansion is similar, except that it is relevant for liquids and gases as well as solids:

\[ \Delta V = \beta V_0 \Delta T, \]

Here, \( \beta \) is the coefficient of volume expansion.
Thermal Expansion (H$_2$O)

Water behaves differently from most other solids—it’s minimum volume occurs when its temperature is 4°C. As it cools further, it expands, as anyone who has left a bottle in the freezer to cool and then forgets about it can testify.
A material may be fixed at its ends and therefore be unable to expand when the temperature changes. It will then experience large compressive or tensile stress—thermal stress—when its temperature changes.

The force required to keep the material from expanding is given by:

\[
\Delta \ell = \frac{1}{E} \frac{F}{A} \ell_0,
\]

where \( E \) is the Young’s modulus (stress/strain) of the material. Therefore, the stress is:

\[
\frac{F}{A} = \alpha E \Delta T.
\]
Absolute Temperature

The concept of absolute zero allows us to define a third temperature scale—the absolute, or Kelvin, scale.

This scale starts with 0 K at absolute zero, but otherwise is the same as the Celsius scale \( \Delta T_C = \Delta T_K \).

Therefore, the freezing point of water is 273.15 K, and the boiling point is 373.15 K \( \Delta T = 100^\circ C = 100K \).
Heat As Energy Transfer

Definition of heat:

Heat is energy transferred from one object to another because of a difference in temperature.
Internal Energy

The sum total of all the energy of all the molecules in a substance is its internal (or thermal) energy.

**Temperature**: measures molecules’ average kinetic energy

**Internal energy**: total energy of all molecules

**Heat**: transfer of energy due to difference in temperature
Specific Heat

The amount of heat \( Q \) required to change the temperature of a material is proportional to the mass and to the temperature change:

\[ Q = mc\Delta T \]

The specific heat, \( c \), is characteristic of the material. Some values are listed at left.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Specific Heat, ( c )</th>
<th>kcal/kg ( \cdot ) C° (( = ) cal/g ( \cdot ) C°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>900</td>
<td>0.22</td>
</tr>
<tr>
<td>Alcohol (ethyl)</td>
<td>2400</td>
<td>0.58</td>
</tr>
<tr>
<td>Copper</td>
<td>390</td>
<td>0.093</td>
</tr>
<tr>
<td>Glass</td>
<td>840</td>
<td>0.20</td>
</tr>
<tr>
<td>Iron or steel</td>
<td>450</td>
<td>0.11</td>
</tr>
<tr>
<td>Lead</td>
<td>130</td>
<td>0.031</td>
</tr>
<tr>
<td>Marble</td>
<td>860</td>
<td>0.21</td>
</tr>
<tr>
<td>Mercury</td>
<td>140</td>
<td>0.033</td>
</tr>
<tr>
<td>Silver</td>
<td>230</td>
<td>0.056</td>
</tr>
<tr>
<td>Wood</td>
<td>1700</td>
<td>0.4</td>
</tr>
<tr>
<td>Ice (−5°C)</td>
<td>2100</td>
<td>0.50</td>
</tr>
<tr>
<td>Liquid (15°C)</td>
<td>4186</td>
<td>1.00</td>
</tr>
<tr>
<td>Steam (110°C)</td>
<td>2010</td>
<td>0.48</td>
</tr>
<tr>
<td>Human body (average)</td>
<td>3470</td>
<td>0.83</td>
</tr>
<tr>
<td>Protein</td>
<td>1700</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Specific heats of gases are more complicated, and are generally measured at constant pressure ($c_p$) or constant volume ($c_v$).
Calorimetry

Closed system: no mass enters or leaves, but energy may be exchanged

Open system: mass may transfer as well

Isolated system: closed system where no energy in any form is transferred

For an isolated system,

Energy out of one part = energy into another part

heat lost = heat gained (Conservation of Energy)
The instrument to the left is a calorimeter, which makes quantitative measurements of heat exchange.

A sample is heated to a well-measured high temperature, plunged into the water, and the equilibrium temperature measured. This gives the specific heat of the sample.
Another type of calorimeter is called a **bomb calorimeter**; it measures the **thermal energy** released when a substance burns.

This is the way the caloric content of foods is measured.