Lecture 7, 7/24/2017

- Faraday’s Law of Induction; Lenz’s Law
- EMF Induced in a Moving Conductor
- Changing Magnetic Flux Produces an Electric Field
- Electric Generators, Back EMF and Counter Torque; Eddy Currents
- Transformers and Transmission of Power
- Inductance
Almost 200 years ago, Faraday looked for evidence that a magnetic field would induce an electric current with this apparatus:
He found no evidence when the current through the left-hand loop was steady, but did see a current induced in the right-hand loop when the switch was turned on or off.
Induced EMF

In addition, a current will be induced in a wire loop if a magnet is moved through the loop, but not when the magnet is held steady.
Therefore, a changing magnetic field induces an emf (a potential difference).

Faraday’s experiment used a magnetic field that was changing because the current producing it was changing; the previous graphic shows a magnetic field that is changing because the magnet is moving.
Faraday’s Law of Induction; Lenz’s Law

The induced emf in a wire loop is proportional to the rate of change of magnetic flux through the loop.

Review of flux

Magnetic flux: $\Phi_B = B \perp A = BA \cos \theta$.

Unit of magnetic flux: weber, Wb.

$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$
Faraday’s Law of Induction; Lenz’s Law

This drawing shows the variables in the flux equation:
Faraday’s Law of Induction; Lenz’s Law

The magnetic flux is analogous to the electric flux—it is proportional to the total number of lines passing through the loop.

\[
\begin{align*}
\theta = 90^\circ & \quad \Phi_B = 0 \\
\theta = 45^\circ & \quad \Phi_B = BA \cos 45^\circ \\
\theta = 0^\circ & \quad \Phi_B = BA
\end{align*}
\]
Faraday’s Law of Induction; Lenz’s Law

Faraday’s law of induction:

\[ E = -\frac{\Delta \Phi_B}{\Delta t}. \]

\[ E = -N \frac{\Delta \Phi_B}{\Delta t}. \]
Faraday’s Law of Induction;
Lenz’s Law

The minus sign gives the direction of the induced emf:

A current produced by an induced emf moves in a direction so that the magnetic field it produces tends to restore the changed field.
Faraday’s Law of Induction; Lenz’s Law

Magnetic flux will change if the area of the loop changes:

(a) (inward)

(b) Flux through coil is decreased because $A$ decreased
Magnetic flux will change if the angle between the loop and the field changes:

(a) Maximum flux
(b) Zero flux
EMF Induced in a Moving Conductor

This image shows another way the magnetic flux can change:
EMF Induced in a Moving Conductor

The induced current is in a direction that tends to slow the moving bar—it will take an external force to keep it moving.
EMF Induced in a Moving Conductor

The induced emf has magnitude

\[ \mathcal{E} = \frac{\Delta \Phi_B}{\Delta t} = \frac{B \Delta A}{\Delta t} = \frac{B \ell v \Delta t}{\Delta t} = B \ell v. \]

Measurement of blood velocity from induced emf:

![Diagram of magnetic field and blood flow](image_url)
Changing Magnetic Flux Produces an Electric Field

A changing magnetic flux induces an electric field; this is a generalization of Faraday’s law.

The electric field will exist regardless of whether there are any conductors around.
A generator is the opposite of a motor—it transforms mechanical energy into electrical energy. This is an ac generator:

The axle is rotated by an external force such as falling water or steam. The brushes are in constant electrical contact with the slip rings.
Electric Generators

A dc generator is similar, except that it has a split-ring commutator instead of slip rings.
Electric Generators

A sinusoidal emf is induced in the rotating loop ($N$ is the number of turns, and $A$ the area of the loop):

$$\mathcal{E} = NB\omega A \sin \omega t$$
Back EMF and Counter Torque

An electric motor turns because there is a torque on it due to the current.

We would expect the motor to accelerate unless there is some sort of drag torque.

That drag torque exists, and is due to the induced emf, called a back emf.
Back EMF and Counter Torque

A similar effect occurs in a generator—if it is connected to a circuit, current will flow in it, and will produce a counter torque. This means the external applied torque must increase to keep the generator turning.
Eddy Currents

Induced currents can flow in bulk material as well as through wires. These are called eddy currents, and can dramatically slow a conductor moving into or out of a magnetic field.
Transformers and Transmission of Power

A transformer consists of two coils, either interwoven or linked by an iron core. A changing emf in one induces an emf in the other.

The ratio of the emfs is equal to the ratio of the number of turns in each coil:

\[
\frac{V_S}{V_P} = \frac{N_S}{N_P}.
\]
Transformers and Transmission of Power

This is a step-up transformer—the emf in the secondary coil is larger than the emf in the primary:

![Diagram of a step-up transformer with primary coil, secondary coil, laminated iron core, and notation for turns and voltages.]
Transformers and Transmission of Power

Energy must be conserved; therefore, in the absence of losses, the ratio of the currents must be the inverse of the ratio of turns:

\[ \frac{I_S}{I_P} = \frac{N_P}{N_S}. \]
Transformers and Transmission of Power

Transformers work only if the current is changing; this is one reason why electricity is transmitted as ac.
Mutual Inductance

Mutual inductance: a changing current in one coil will induce a current in a second coil.

\[ \mathcal{E}_2 = -M \frac{\Delta I_1}{\Delta t} \]

And vice versa; note that the constant \( M \), known as the mutual inductance, is the same:

\[ \mathcal{E}_1 = -M \frac{\Delta I_2}{\Delta t} \]
Inductance

Unit of inductance: the henry, H.

\[ 1 \text{ H} = 1 \text{ V} \cdot \text{s/} \text{A} = 1 \text{ } \Omega \cdot \text{s} \]

A transformer is an example of mutual inductance.